

Leveraging Optimized FFT on Intel Platforms

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Agenda

- Discrete Fourier Transform in applications
- Fast Fourier Transform (FFT) in a nutshell
- FFTW & MKL DFT
- 3D FFT in applications
- Summary

6

Discrete Fourier transforms (DFT)



DFT in Applications

- Signal processing, spectroscopy, magnetic resonance imaging,
- Speech and image recognition
- Simulations of periodic systems
- Quantum computing

"It's hard to explain what sort of impact Fourier's had," because the Fourier transform is such a fundamental concept that by now, "it's part of the language," Demanet, http://news.mit.edu/2009/explained-fourier





Fourier transform as matrix-vector product: $O(N^2)$

$$x_{k} = (1/N) \sum_{n=0}^{N-1} e^{+2\pi i k n/N} y_{n}$$

$$\omega = e^{-2\pi i/N} = \cos \frac{2\pi}{N} - i \sin \frac{2\pi}{N}$$

$$y_{k} = \sum_{n=0}^{N-1} e^{-2\pi i k n/N} x_{n}$$

$$y = \left(\omega^{jk} \right) x$$

Fast Fourier Transform (FFT): a fast way to compute DFT

Fast Fourier Transformation (FFT), *O(N log(N))* The Cooley-Tukey algorithm

8-point FFT

$$FFT8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & -i & -i\omega & -1 & -\omega & i & i\omega \\ 1 & -i & -1 & i & 1 & -i & -1 & i \\ 1 & -i\omega & i & \omega & -1 & i\omega & -i & -\omega \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\omega & -i & i\omega & -1 & \omega & i & -i\omega \\ 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & i\omega & i & -\omega & -1 & -i\omega & -i & \omega \end{bmatrix}$$

 $W = \omega = e^{-2\pi i/8} = (1 - i)/\sqrt{2}$ Twiddle factors: $W^0, W^1, ..., W^7$



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Typical Ways to Compute FFT

Cooley-Tukey

Recursively split the FFT into smaller equal sizes

Split-Radix

Radix 4:2:2

Stockham

Eliminates the need for rearranging the inputs/outputs that is specific to Cooley-Tukey

Prime-factor

Decompose into relatively prime numbers

Bluestein (Chirp-Z)

 For arbitrary sizes, uses Cooley-Tukey and convolution theorem many others...

Basic 3D FFT using stride-1 1D FFT

$$\hat{f}(k_x, k_y, k_z) = \sum_x \sum_y \sum_z e^{i2\pi (k_x x + k_y y + k_z z)} f(x, y, z) = \sum_x e^{i2\pi k_x x} \left[\sum_y e^{i2\pi k_y y} \left[\sum_z e^{i2\pi k_z z} f(x, y, z) \right] \right]$$



- Optimized 1D FFT on stride-1 for each direction
- Transpose to transform the data for 1D FFT

We have libraries for FFT

- MKL-FFT, FFTW ...
- Highly optimized 1D FFT $2^p 3^q 5^r \cdots P^z$
- Optimized N-dim FFT and transposes
- Building blocks for DIY FFT

How to use FFT libraries to maximize the productivity!

FFTW and MKL DFT

- Both implement 1 to multi-dimensional FFT
 - Complex-to-Complex
 - Complex-to-Real and Real-to-Complex
 - Double & single precision
 - Threaded on SMP nodes and distributed FFT on multi SMP nodes
- FFTW <u>www.fftw.org</u>
 - Introduced FFT plan concepts
 - Novel code-generation and runtime self-optimization techniques
- Intel[®] MKL Fast Fourier Transforms (next)
 - Highly optimized on Intel platforms
 - Provide FFTW Wrapper

Intel[®] MKL Fast Fourier Transforms (FFTs)

FFTW Interfaces support	C, C++ and FORTRAN source code wrappers provided for FFTW2 and FFTW3. FFTW3 wrappers are already built into the library		
Cluster FFT -	Perform Fast Fourier Transforms on a cluster		
	Interface similar to DFTI		
	Multiple MPIs supported		
Parallelization	Thread safe with automatic thread selection		
Storage Formats	Multiple storage formats such as CCS, PACK and Perm supported		
Batch support	Perform multiple transforms in a single call		
Additional - Features -	Perform FFTs on partial images		
	Padding added for better performance		
	Transform combined with transposition		
	mixed-language usage supported		

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Intel® MKL 2018 New FFT Features & Optimizations

MKL_VERBOSE support for FFT starting from MKL 2018 Update 1

set MKL_VERBOSE=1. The verbose information in the program will be shown up: MKL_VERBOSE Intel(R) MKL 2018.0 Update 1 Product build 20170712 for Intel(R) 64 architecture Intel(R) Advanced Vector Extensions 2 (Intel(R) AVX2) enabled processors, Lnx 2.30GHz gnu_thread NMICDev:0 MKL_VERBOSE FFT: MAIN_DESC | sro4:5:3x5:1:1 | THR_LIMIT = 1 | 0.00s CNR:OFF Dyn:1 FastMM:1 TID:0 NThr:36 WDiv:HOST:+0.000

 Significantly improved 1D and 3D FFT performance for Intel[®] Xeon[®] Processor supporting Intel[®] Advanced Vector Extensions 512 (Intel[®] AVX-512)
 https://software.intel.com/on.us/mkl/features/henchmarks

https://software.intel.com/en-us/mkl/features/benchmarks

Typical FFT use in applications

FFTW APIs

void do_fftw(){

const int N=3; int ngrid[]={64,64,64}; const int howmany=4; const size_t distance=64*64*64; fftw_complex in[howmany*distance]; fftw_complex out[howmany*distance];

//FFT
fftw_execute_dft(for_plan,in,out);

//inverse FFT
fftw_execute_dft(back_plan,out,in);

```
fftw_destroy_plan(back_plan);
fftw_destroy_plan(for_plan);
```

1. Create plans for forward FFT and inverse FFT

- Require addresses of In/Out
- Select (FFTW_ESTIMATE, FFTW_MEASURE, FFTW_ PATIENT,..)
- Alternative APIs
- 2. Execute Forward FFT
- 3. Execute backward (Inverse) FFT
- 4. Cleanup

Using MKL DFT APIs

void do_mkl_dfti(){

```
const int N=3;
int ngrid[]={64,64,64};
const int howmany=4;
const size_t distance=64*64*64;
_MKL_Complex16 in[howmany*distance];
_MKL_Complex16 out[howmany*distance];
```

```
DFTI_DESCRIPTOR_HANDLE mkl_plan;
```

```
MKL_LONG status = DftiCreateDescriptor(&mkl_plan,
        DFTI_DOUBLE,DFTI_COMPLEX,N,ngrid);
status=DftiSetValue(mkl_plan,DFTI_NUMBER_OF_TRANSFORMS,howmany);
status=DftiSetValue(mkl_plan,DFTI_INPUT_DISTANCE,distance);
DftiCommitDescriptor(mkl_plan);
```

//FFT

```
DftiComputeForward(mkl_plan,in,out);
```

//inverse FFT

DftiComputeBackward(mkl_plan,out,in);

```
DftiFreeDescriptor(&mkl_plan);
```

- 1. Create a descriptor
- 2. Set configuration parameters (optional)
- 3. Commit a descriptor
- 4. Execute Forward FFT
- 5. Execute backward (Inverse) FFT
- 6. Cleanup

FFTW Wrapper in MKL

- No code modification necessary.
- Include the header file: -I\${MKLROOT}/include/fftw
- Link MKL as usual, e.g., -mkl

Mapping between FFTW and Intel MKL Interfaces

https://software.intel.com/en-us/articles/the-intel-math-kernel-library-andits-fast-fourier-transform-routines

3D FFT in electronic structure codes

PW DFT on Parallel Computers Clarke, Stich & Payne, CPC, 72, 14 (1992)

Computer Physics Communications 72 (1992) 14–28 North-Holland Computer Physics Communications

Large-scale ab initio total energy calculations on parallel computers

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I. Štich¹ and M.C. Payne Cavendish Laboratory (TCM), University of Cambridge, Madingley Road, Cambridge, CB3 0HE, UK

Received 30 March 1992

An implementation of a set of total energy plane-wave pseudopotential codes on a parallel computer is described which allows calculations to be performed for systems containing many hundreds of atoms in the unit cell. Possible parallelisation strategies are discussed and it is shown that assigning parts of real and Fourier space across the processors is the least restricted approach. The performance of our parallel codes is demonstrated by timing tests carried out on several i860-based parallel machines and these are compared with tests performed on conventional sequential supercomputers. Ab initio computations on systems which are beyond the power of conventional supercomputers as well as new perspectives for first-principles molecular dynamics are discussed. Abinit BigDFT CASTEP CPMD GPAW NWChem-PW Qbox Quantum Espresso VASP

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Algorithm 8: A pseudo PW KS-DFT code



C(Ng, N), the solution Ng=Nx * Ny * Nz, FFT grid N= number of KS orbials

Data parallelization (FORTRAN convention)

Band distribution

- Distribute M
- FFT within a task; exploit optimized FFT library
- Choice for HF, GW & RPA



FFT distribution

- Distribute Ng
- Cluster FFT
- Choice for DFT in many codes



Mixed

- FFT groups
- Band groups



Band & FFT distribution



Distribute M and Ng Distribute FFT Parallel GEMM (aka Scalapack)

- Implemented in VASP, Qbox, NWCHEM-PW
- Necessary for small-memory systems (e.g., Blue Gene L/P/Q): Qbox 2006 Gordon Bell
- Maybe needed for big problems
- May provide the best compromise between FFT and GEMM efficiency
- Can leverage TSGEMM and TSQR

Optimizing 3D FFT use

- Many concurrent FFTs of Nx*Ny*Nz
 - M=10-1000
 - FFT grid Nx=10-200 for typical problems of today
- Memory bandwidth or network bandwidth limited
- Cache blocking, SIMD and overlap of the data movement and computation: critical for high performance
- Leverage "many (batched)" FFT
- Moving data efficiently via optimized transpose
- Overlap with other computations

FFT 3D on SMP: OpenMP vs MPI GLOPS=Theoretical Cooley-Tukey



- Using 64 cores of 72-core Intel(R) Xeon Phi(TM) CPU 7250 @ 1.40GHz
- I_MPI_PIN_DOMAIN= 64*4/MPI
- KMP_AFFINITY=(scatter,balanced,compact), granularity=fine



- Optimize either throughputs (how many FFT/s) or time (strong scaling)
- Using 64 cores of 72-core Intel(R) Xeon Phi(TM) CPU 7250 @ 1.40GHz
- I_MPI_PIN_DOMAIN= 64*4/MPI; KMP_AFFINITY=(scatter,balanced,compact), granularity=fine

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Application-tailored FFT

- Data structure and distribution set by the entire application
- Can interleave FFT and other computations
- Can reduce data movement
- Can overlap communications/computations

2D FFT example Transpose 1D FFT Transpose 1D FFT In-place transpose

Using MKL FFT configuration parameters for customized FFT: composed 3D FFT=2D + 1D

MKL_LONG ngrid_2d[]={ngrid[1],ngrid[2]};

MKL LONG strides[2]={0,ngrid[1]*ngrid[2]};

Create 2D FFT plan

DFTI_DESCRIPTOR_HANDLE plan_yz, plan_x; MKL_LONG status = DftiCreateDescriptor(&plan_yz,dfti_get_precision(real_type()),DFTI_COMPLEX,2,ngrid_2d); status=DftiSetValue(plan_yz,DFTI_NUMBER_OF_TRANSFORMS,ngrid[0]); status=DftiSetValue(plan_yz,DFTI_INPUT_DISTANCE,ngrid[1]*ngrid[2]); DftiCommitDescriptor(plan_yz);

Create 1D FFT plan

status=DftiCreateDescriptor(&plan_x,dfti_get_precision(real_type()),DFTI_COMPLEX,1,ngrid[0]); status=DftiSetValue(plan_x,DFTI_NUMBER_OF_TRANSFORMS,ngrid[1]*ngrid[2]); status=DftiSetValue(plan_x,DFTI_INPUT_DISTANCE,1); status=DftiSetValue(plan_x,DFTI_OUTPUT_DISTANCE,1); status=DftiSetValue(plan_x,DFTI_INPUT_STRIDES,strides); status=DftiSetValue(plan_x,DFTI_OUTPUT_STRIDES,strides); DftiCommitDescriptor(plan_x);

DftiComputeForward(plan_yz,g[0]); DftiComputeForward(plan_x,g[0]);

DftiComputeBackward(plan_x,g[0]); DftiComputeBackward(plan_yz,g[0]); Backward 3D FFT

Forward 3D FFT

https://software.intel.com/en-us/mkl-developer-reference-c-configuration-settings



29

Performance of "many" 3D FFT of 128³

GFLOPS, higher the better



of FFT = # of MPI ranks
Theoretical Cooley-Tukey GFLOPS

Intel[®] Xeon[®] Platinum 8170 CPU, 2S, 26C/S

Reducing BW needs

- Multiple threads for each FFT : SMP optimization and no MPI communications
- Cache/register optimization via in-place, on-the-fly transpose and SIMD optimization in MKL
- A FFT is 80 times faster using Composed methods using 26 threads than the baseline using 1 thread.



- Overview of FFT capabilities in MKL
- Considerations to maximize FFT performance in applications
 - Allocation
 - Computation
 - Data movement
- Adapt FFT use for application-specific data structures and algorithms based on FFT primitives

